**Quiz number ( 1 ) Mathematic 2B**

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|---|---|

......... Use power series to solve the equation $y'' + xy' + y = 0$

**Answer**

We assume there is a solution of the form $y = \sum_{n=0}^{\infty} a_n x^n$

so $y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$ and $y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$

Substitute in the equation $\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=1}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} a_n x^n = 0$

Let the first series start from 0 $\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n + \sum_{n=1}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} a_n x^n = 0$

Separate first term from the first and third series to both start from 1

$$2a_2 + a_0 + \sum_{n=1}^{\infty} (n+2)(n+1) a_{n+2} x^n + \sum_{n=1}^{\infty} n a_n x^n + \sum_{n=1}^{\infty} a_n x^n = 0$$

Now collect the series $2a_2 + a_0 + \sum_{n=1}^{\infty} \left[(n+2)(n+1) a_{n+2} + (n+1) a_n\right] x^n = 0$

Then we have

$$2a_2 + a_0 = 0 \rightarrow a_2 = -\frac{a_0}{2}$$

$$\left[(n+2)(n+1) a_{n+2} + (n+1) a_n\right], \; n = 1, 2, 3, \ldots$$

$$a_{n+2} = \frac{-(n+1)}{(n+1)(n+2)} a_n = \frac{-a_n}{(n+2)}, \; n = 1, 2, 3, \ldots \qquad (1)$$

We solve this recursion relation by putting successively in Equation 1

Put $n = 1:$ $\quad a_3 = \frac{-a_1}{3}$

Put $n = 2:$ $\quad a_4 = \frac{-1}{4} a_2 = \frac{1}{2.4} a_0$

Put $n=3$: $a_5=\frac{-1}{5}a_3=\frac{a_1}{3.5}$

Put $n=4$: $a_6=\frac{-1}{6}a_4=\frac{-1}{2.4.6}a_0$

Put $n=5$: $a_7=\frac{-1}{7}a_5=\frac{-a_1}{3.5.7}$

Put $n=6$: $a_8=\frac{-1}{8}a_6=\frac{1}{2.4.6.8}a_0$

Put $n=7$: $a_9=\frac{-1}{9}a_7=\frac{a_1}{3.5.7.9}$

In general, the even coefficients are given by $a_{2k}=\frac{(-1)^{k-1}}{2.4.6...(2k)}a_0=\frac{(-1)^{k-1}}{2^k(k)!}a_0$

And the odd coefficients are given by $a_{2k+1}=\frac{(-1)^k}{3.5.7...(2k+1)}a_1=\frac{(-1)^k 2^k k!}{(2k+1)!}a_1$

The solution is

$$y=a_0+a_1x-\frac{x^2}{2}a_0-\frac{x^3}{3}a_1+\frac{x^4}{2.4}a_0+\frac{x^5}{3.5}a_1-\frac{x^6}{2.4.6}a_0-\frac{x^7}{3.5.7}a_1+\frac{x^8}{2.4.6.8}a_0+\frac{x^9}{3.5.7.9}a_1$$

$$\boxed{y=a_0\left[1-\frac{x^2}{2}+\frac{x^4}{2.4}-\frac{x^6}{2.4.6}+\frac{x^8}{2.4.6.8}+...\right]+a_1\left[x-\frac{x^3}{3}+\frac{x^5}{3.5}-\frac{x^7}{3.5.7}+\frac{x^9}{3.5.7.9}+...\right]}$$

$$\boxed{y=a_0\left(1+\sum_{k=1}^{\infty}\frac{(-1)^k}{2^k(k)!}x^{2k}\right)+a_1\left(x+\sum_{k=1}^{\infty}\frac{(-1)^k 2^k k!}{(2k+1)!}x^{2k+1}\right)}$$

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$$a_{n+2}=\frac{-(n+1)}{(n+1)(n+2)}a_n \qquad n=1,2,3,... \qquad (1)$$

Put $n=1$: $a_3=\frac{-2a_1}{2.3}$

Put $n=2$: $a_4=\frac{-3}{3.4}a_2=\frac{3}{2.3.4}a_0$

Put $\quad n=3: \quad a_5=\frac{-4}{4.5}a_3=\frac{2.4}{2.3.4.5}a_1$

Put $\quad n=4: \quad a_6=\frac{-5}{5.6}a_4=\frac{-3.5}{2.3.4.5.6}a_0$

Put $\quad n=5: \quad a_7=\frac{-6}{6.7}a_5=\frac{-2.4.6}{2.3.4.5.6.7}a_1$

In general, the even coefficients are given by

$$a_{2k}=\frac{(-1)^{k-1}[1.3.5....(2k-1)]}{1.2.3.4.....(2k)}a_0=\frac{(-1)^{k-1}[1.3.5....(2k-1)]}{(2k)!}a_0$$

And the odd coefficients are given by $a_{2k+1}=\frac{(-1)^k[2.4.6...(2k)]}{2.3.4.5....(2k+1)}a_1=\frac{(-1)^k 2^k k!}{(2k+1)!}a_1$

The solution is

$$y=a_0+a_1x-\frac{x^2}{2}a_0-\frac{2x^3}{2.3}a_1+\frac{3x^4}{2.3.4}a_0+\frac{2.4x^5}{3.4.5}a_1-\frac{3.5x^6}{2.3.4.5.6}a_0-\frac{2.4.6x^7}{3.5.7}a_1+...$$

$$y=a_0\left[1-\frac{x^2}{2}+\frac{3x^4}{2.3.4}-\frac{3.5x^6}{2.3.4.5.6}+...\right]+a_1\left[x-\frac{2x^3}{2.3}+\frac{2.4x^5}{3.4.5}-\frac{2.4.6x^7}{2.3.4.5.7}+...\right]$$

والتى يمكن تبسيطها على الصورة

$$y=a_0\left[1-\frac{x^2}{2!}+\frac{3x^4}{4!}-\frac{3.5x^6}{6!}+...\right]+a_1\left[x-\frac{2x^3}{3!}+\frac{2.4x^5}{5!}-\frac{2.4.6x^7}{7!}+...\right]$$

وتوضع بالصورة

$$y=a_0\left(1+\sum_{k=1}^{\infty}\frac{(-1)^k[1.3.5....(2k-1)]}{(2k)!}x^{2k}\right)+a_1\left(x+\sum_{k=1}^{\infty}\frac{(-1)^k 2^k k!}{(2k+1)!}x^{2k+1}\right)$$

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$$y=a_0\left(1+\sum_{k=1}^{\infty}\frac{(-1)^k}{2^k k!}x^{2k}\right)+a_1\left(x+\sum_{k=1}^{\infty}\frac{(-1)^k 2^k k!}{(2k+1)!}x^{2k+1}\right)$$

Or $\quad y=a_0\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{2^k k!}x^{2k}\right)+a_1\left(\sum_{k=0}^{\infty}\frac{(-1)^k 2^k k!}{(2k+1)!}x^{2k+1}\right)$

إذا توجد صور متعددة للإجابة النهائية سواء الحدود او بصيغة الحد العام